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# Sparse Multi-Carrier Index Keying OFDM with Index Separation over Correlated Sub-Carriers

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**Abstract**—In this paper, we propose a sparse multi-carrier index keying (MCIK) method for orthogonal frequency division multiplexing (OFDM) system, which uses the indices of sparse sub-carriers to transmit the data, and improve the performance of signal detection in highly correlated sub-carriers. Although a receiver is able to exploit a power gain with precoding in OFDM, the sensitivity of the signal detection is usually high as the orthogonality is not retained in highly dispersive environments. To overcome this, we focus on developing the trade-off between the sparsity of the MCIK, correlation, and performances, analyzing the average probability of the error propagation imposed by incorrect index detection over highly correlated sub-carriers. In asymptotic cases, we are able to see how sparsity of MCIK should be designed in order to perform superior to the classical OFDM system. Based on this feature, sparse MCIK based OFDM is a better choice for low detection errors in highly correlated sub-carriers.

**Index Terms**—Sparse multi-carrier index keying, non-orthogonal OFDM, detection error propagation.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted in the majority of today and future communication standards such as IEEE 802.11, 3GPP's LTE-Advanced, due to its robustness to multipath fading. The performance of these systems with increased bandwidth is heavily dependent on an increased sensitivity to non-orthogonality of sub-carriers due to frequency offset and Doppler shift [1] as well as transmission nonlinearity caused by the non-constant power ratio of OFDM symbols [2], [3].

In [4], [5], the so-called sub-carrier-index modulation (SIM) OFDM scheme has been proposed to outperform the classical OFDM systems. The sub-carrier index becomes additional resource to decrease the bit error rate (BER) faster than the classical OFDM with only a few sub-carrier activation. Recently, the effects of channel estimation errors on the approximate pairwise error probability (PEP) of the OFDM modulating the index of sub-carrier was discussed in [6]. OFDM modulating the indices of active sub-carriers are more recently combined with space-time block coding [7] and with compressive sensing [8], for better performance. They are evaluated based on orthogonal sub-carriers. In practice, however, orthogonality of sub-carriers at the receiver is hard to retain, especially in frequency dispersive channels. For the robustness to the non-orthogonality of sub-carriers, precoding

can be used in SIM OFDM. In this case, the detection error probability can still be high as the orthogonality is not retained.

In this work, we propose SIM for precoded OFDM with a small number of activated sub-carriers which are designed to be apart over correlated sub-carriers. Since only a fraction of sub-carriers are activated, the resulting modulation is referred to as sparse multi-carrier index keying (MCIK). The contribution of this paper is three-fold. We first analyze the propagation error rate (PER) expression of sparse MCIK for any number of active sub-carriers and secondly investigate the effect of propagation of error imposed by the nature of sparse MCIK in highly correlated sub-carriers. In [4], [5], the BER is limited by a fixed number of active, orthogonal sub-carriers that differs from what we consider herein. For example, the approach in [5] cannot be used directly with either the small or large active sub-carriers as well as non-orthogonal sub-carriers. Our contribution is thirdly to develop the trade-off between the sparsity of the MCIK, correlation, and PER at various extreme cases. In the presence of highly correlated sub-carriers, it aims to provide theoretical guidelines with the fundamental question: how sparsity of the MCIK OFDM should be chosen along with the maximum index separation to decrease the detection error rate, outperforming the classical OFDM.

## II. SYSTEM MODEL

We consider an OFDM transmission with  $L$  sub-carriers. Denote by  $\mathbf{s}$  an OFDM signal block and its elements are referred to as data symbols, for convenience. The received signal in the frequency domain is given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n},$$

where  $\mathbf{H} = \text{diag}(H_1, \dots, H_L)$  is a diagonal channel matrix,  $H_i$  represent Rayleigh fading channel coefficients observed in subcarrier  $i$ , being complex Gaussian with zero mean and unit variance, i.e.,  $H_i \sim \mathcal{CN}(0, 1), \forall i$ , and  $\mathbf{n}$  is the independent, additive white Gaussian noise (AWGN) vector, i.e.,  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ .

In practice, we face frequency dispersive channels, due to the presence of, e.g., phase noise and frequency offsets. That is,  $H_i, \forall i$  are non-independent, and identically distributed leading to a heterogeneous cross correlation coefficient:  $\rho_{i,j} =$

$\mathbb{E}(H_i H_j^*) \in [0, 1)$ , where we have  $\rho_{i,j} \neq \rho_{l,m}$  and  $\rho_{i,j} = \rho_{j,i}, \forall i, j, l, m$ .

In the classical OFDM system, each data symbol is an element of a given signal constellation. That is,  $s_l \in \mathcal{S}$ , where  $s_l$  is the  $l$ th element of  $\mathbf{s}$  and  $\mathcal{S}$  represents the signal constellation. Using the maximum likelihood (ML) detector,  $s_l$  can be detected from the  $l$ th element of  $\mathbf{y}$ , i.e.,  $y_l$ , as

$$\hat{s}_l = \arg \min_{s_l \in \mathcal{S}} |y_l - H_l s_l|^2.$$

### III. SPARSE MULTI-CARRIER INDEX KEYING OFDM WITH INDEX SEPARATION

Let  $L$  sub-carriers be composed of  $K$  clusters, each containing  $N$  sub-carriers (i.e.,  $L = KN$ ). Denote by  $\mathbf{W}$  the precoding matrix and by  $\mathbf{x}$  the  $K$  non-zero data symbols. The received signal can be

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{W}\mathbf{x} + \mathbf{n}, \\ &= \mathbf{H}\mathbf{s} + \mathbf{n}, \end{aligned} \quad (1)$$

where  $\mathbf{s} = \mathbf{W}\mathbf{x}$  is the precoded OFDM signal block,  $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_K]$ , its  $k$ th column vector has  $\|\mathbf{w}_k\|_0 = 1, \forall k$ . For every transmission,  $\mathbf{s}$  is used to modulate sub-carriers, as in the classical OFDM system, but it differs from that the modulated sub-carriers are only a fraction of sub-carriers. That is, at cluster  $k$   $\mathbf{w}_k$  decides part of sub-carriers to modulate. Unlike the conventional MCIK-OFDM [6], [9] whose sub-carrier indices are assumed to be adjacent to each other, we propose the indices of sub-carriers at cluster  $k$  are separated by  $\alpha_\Delta$ , i.e.,  $\{\alpha_k, \alpha_k + \alpha_\Delta, \dots, \alpha_k + (N-1)\alpha_\Delta\}$ . Thus,  $\alpha_\Delta$  denotes the minimum separation distance between indices of sub-carriers at each cluster.

For MCIK OFDM with the index separation, most  $s_l$ 's are zero, while information can be conveyed by both non-zero data symbols (in  $\mathbf{s}$ ) and indices of  $K$  active sub-carriers. Consider  $K$  non-zero data symbols in  $\mathbf{s}$ , i.e.,  $\mathbf{s}$  is a  $K$ -sparse signal vector, which leads to  $\|\mathbf{s}\|_0 = K$ . Given  $L$ ,  $\alpha_\Delta$  is a design parameter as an integer, i.e.,  $\alpha_\Delta \in \{1, \dots, K\}$ . In other words, the proposed scheme can benefit from properly designing both  $\alpha_\Delta$  and  $K$ , over non-orthogonal sub-carrier environments. We denote the average signal-to-noise ratio (SNR) by  $\bar{\gamma} = \tau E_s / N_0$  where  $E_s$  denotes the average power for non-zero data symbol and  $\tau$  represents the  $K$ -sparsity ratio, i.e.,  $\tau = L/K$ .

Denote by  $m_I$  the number of information bits that are conveyed through the indices of  $K$  sub-carriers. This is given by

$$m_I = \lfloor \log_2 B(L, K) \rfloor,$$

where  $B(\cdot)$  denotes the binomial coefficient. Denote by  $m_D$  the number of information bits that can be sent by the non-zero data symbols. That is  $m_D = K \lfloor \log_2 |\mathcal{S}| \rfloor$  [9]. For example, when  $L = 64$  and  $K = 6$ , we have  $m_I = 26$  bits and each non-zero data symbol can represent  $\log_2 |\mathcal{S}|$  bits. Thus, the total number of transmit bits is

$$R = \lfloor \log_2 B(L, K) \rfloor + K \log_2 |\mathcal{S}|, \quad (2)$$

when  $|\mathcal{S}|$  is a power of 2 such as  $|\mathcal{S}| = M$  for M-QAM.

### IV. ERROR PROPAGATION ANALYSIS

We consider two step approach for detection. The ML detection can be considered to detect  $\mathbf{s}$  in MCIK, and the indices of non-zero elements of the estimated  $\mathbf{s}$  are used to detect  $m_I$ . Denote  $\tilde{\mathcal{S}} = \mathcal{S} \cup \{0\}$ , which is the extended signal constellation of  $\mathcal{S}$  including zero. Based on this, the ML detection can be made as

$$\begin{aligned} \hat{\mathbf{s}} &= \arg \min_{\mathbf{s}} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \\ \text{subject to } s_l &\in \tilde{\mathcal{S}} \text{ and } \mathbf{s} \in \Sigma_K, \end{aligned} \quad (3)$$

where  $\Sigma_K$  is a set of possible  $K$ -sparse signals, which is defined as

$$\Sigma_K = \{ \mathbf{s} \mid \|\mathbf{s}\|_0 = K \} \quad (4)$$

and its cardinality is  $|\Sigma_K| = 2^{m_I}$ .

The symbol errors in MCIK result from three error cases [9]:

- (i) an incorrect index of active sub-carrier and an incorrect data symbol;
- (ii) an incorrect index of active sub-carrier and a correct data symbol; and
- (iii) a correct index of active sub-carrier and an incorrect data symbol.

Notice from these error cases that all symbol errors are relying on the accuracy of index detection, which acts an important role in the overall symbol error rate. This observation leads to that inaccuracy of index detection will cause propagation of errors on the detection of non-zero data symbols (for  $m_D$ ) as well as index modulation symbols (for  $m_I$ ). This error propagation is critical to analyze in the MCIK system.

#### A. Average PEP over non-orthogonal sub-carriers

Denote by  $(s_{k,\alpha} \rightarrow s_{k,\tilde{\alpha}})$  the pairwise error event (PEE) that in cluster  $k$  active index  $\alpha$  is incorrectly detected as  $\tilde{\alpha}$  for  $\alpha, \tilde{\alpha} \in \{1, \dots, L\}$  and  $\alpha \neq \tilde{\alpha}$ , given that  $\alpha$  is transmitted within cluster  $k$ . Here, let  $\alpha_\Delta = \min |\alpha - \tilde{\alpha}|$  and  $\mathbf{s}_{k,\alpha} = \mathbf{w}_k x_k$  represent  $\mathbf{s}$  of cluster  $k$  with all zero elements except the  $\alpha$ th element.

Based on the PEE, the symbol error rate ( $P_{s,k}$ ) can be imposed by incorrect index and is obtained, using the union bound, as [9]

$$P_{s,k} \leq \sum_{\alpha} \sum_{\tilde{\alpha} \neq \alpha} P(s_{k,\alpha} \rightarrow s_{k,\tilde{\alpha}}) \frac{1}{N}, \quad (5)$$

where  $P(s_{k,\alpha} \rightarrow s_{k,\tilde{\alpha}})$  is the conditional pairwise error probability (PEP) of deciding  $s_{k,\tilde{\alpha}}$  given that  $s_{k,\alpha}$  and  $H_\alpha$  are used, and the priori probabilities of  $s_{k,\alpha}$  are equally likely.

Using the likelihood ratio test (LRT) in [10], the well-known PEP expression in (5) can be

$$P(s_{k,\alpha} \rightarrow s_{k,\tilde{\alpha}}) = Q \left( \sqrt{\frac{d_k}{4N_0}} \right), \quad (6)$$

where  $Q(z) \triangleq \pi^{-1} \int_0^{\pi/2} e^{-z^2/2 \sin^2 \theta} d\theta$  is the error function and  $d_k = \tau E_s (|H_\alpha|^2 + |H_{\tilde{\alpha}}|^2)$ .

Consider the average PEP with the presence of positive correlation of sub-carriers. Denote by  $z_\alpha |H_\alpha|^2$  and by  $\tilde{z}_\alpha |H_{\tilde{\alpha}}|^2$ , for simple notations. Then, (6) relies on both  $z_\alpha$  and  $\tilde{z}_\alpha$ . The average PEP can be therefore obtained, using a joint probability distribution function (pdf) of  $z_\alpha$  and  $\tilde{z}_\alpha$ , as

$$\int_0^\infty \int_0^\infty e^{-\frac{\tau E_s}{8N_0}(z_\alpha + \tilde{z}_\alpha)} p(\tilde{z}_\alpha | z_\alpha) p(z_\alpha) d\tilde{z}_\alpha dz_\alpha, \quad (7)$$

where  $p(\cdot|\cdot)$  and  $p(\cdot)$  represent the conditional pdf of  $\tilde{z}_\alpha$  on  $z_\alpha$  and the pdf of  $z_\alpha$ , respectively.

Particularly with the proposed system model, we have  $p(z_\alpha) = e^{-z_\alpha}$  and

$$p_{\tilde{z}_\alpha | z_\alpha}(\tilde{z} | z) = \frac{1}{\sigma_\epsilon^2} e^{-\frac{\tilde{z} + \rho_{\tilde{\alpha}, \alpha}^2}{\sigma_\epsilon^2}} I_0 \left( \sqrt{\tilde{z}z} \frac{|-2\rho_{\tilde{\alpha}, \alpha}|}{\sigma_\epsilon^2} \right), \quad (8)$$

where  $\sigma_\epsilon^2 = (1 - \rho_{\tilde{\alpha}, \alpha}^2) \sigma_h^2$  with the normalized sub-carrier's variance  $\sigma_h^2 = 1$  in our system model,  $\rho_{\tilde{\alpha}, \alpha} \in [0, 1)$ ,  $\forall \alpha, \tilde{\alpha}$ , and  $I_0(\cdot)$  stands for the modified bessel function of the first kind.

Using these pdfs and [11, (3.326.3)], we can compute the integrals in (7); for simplicity, detailed computation steps are omitted. Thus, the upper bound expression for the average PEP per cluster  $k$  can be in closed-form as

$$\begin{aligned} \text{aPEP}_k &\leq \frac{A(\bar{\gamma}, \sigma_\epsilon^2)^{-2}}{\pi} \left\{ \sigma_\epsilon^2 + \frac{A(\bar{\gamma}, \sigma_\epsilon^2)^{1/2}}{2} \right. \\ &\quad \times \left. \frac{|-2\rho_{\alpha, \tilde{\alpha}}| \Gamma(3/2)}{\sqrt{\sigma_\epsilon^2}} \left( 1 + \frac{\bar{\gamma}}{8} + \frac{\rho_{\alpha, \tilde{\alpha}}^2}{\sigma_\epsilon^2} \right)^{-3/2} \right\}, \end{aligned} \quad (9)$$

where  $A(\bar{\gamma}, \sigma_\epsilon^2) = 1 + \frac{\bar{\gamma}\sigma_\epsilon^2}{8}$ . To further emphasize the impact of  $\rho_{\alpha, \tilde{\alpha}}$ , this inequality can be rewritten as

$$\begin{aligned} \text{aPEP}_k &\leq \frac{1}{\pi} \left( 1 + \frac{(1 - \rho_{\alpha, \tilde{\alpha}}^2) \bar{\gamma}}{8} \right)^{-2} \left\{ (1 - \rho_{\alpha, \tilde{\alpha}}^2) \right. \\ &\quad \left. + \frac{|-\rho_{\alpha, \tilde{\alpha}}| \Gamma(3/2) (1 - \rho_{\alpha, \tilde{\alpha}}^2)}{1 + (1 - \rho_{\alpha, \tilde{\alpha}}^2) \bar{\gamma}/8} \right\}. \end{aligned} \quad (10)$$

#### B. Total average propagation error rate (APER)

From (5) and (10), the average probability of the error propagation imposed by the incorrect index detection can be obtained over the non-orthogonal sub-carriers and is given:

$$\begin{aligned} P_e(\bar{\gamma}, \Sigma_\rho) &\leq L^{-1} \sum_{k=1}^K \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha}^N \text{aPEP}_k(\bar{\gamma}, \rho_{\alpha, \tilde{\alpha}}) \\ &= \tau^{-1} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha}^N \frac{1}{\pi} \left( 1 + \frac{(1 - \rho_{\alpha, \tilde{\alpha}}^2) \bar{\gamma}}{8} \right)^{-2} \\ &\quad \times \left\{ (1 - \rho_{\alpha, \tilde{\alpha}}^2) + \frac{|-\rho_{\alpha, \tilde{\alpha}}| \Gamma(3/2) (1 - \rho_{\alpha, \tilde{\alpha}}^2)}{1 + (1 - \rho_{\alpha, \tilde{\alpha}}^2) \bar{\gamma}/8} \right\}, \end{aligned} \quad (11)$$

where  $\Sigma_\rho$  denotes a set of the cross correlation:  $\Sigma_\rho = \{\rho_{\alpha, \tilde{\alpha}}\}$ ,  $\forall \alpha, \tilde{\alpha}$ , and recall that  $\bar{\gamma} = \tau E_s / N_0$  and the sparsity ratio  $\tau = L/K$ . It can be observed from (11) that for given  $L$ ,  $P_e(\cdot, \cdot)$  relies on  $K$  as well as  $\rho_{\alpha, \tilde{\alpha}}$ 's.

To clearer view of such relationship between  $\tau (= L/K)$  and  $P_e(\cdot, \cdot)$ , consider an example experiencing correlated sub-carriers with  $\rho_{\alpha, \tilde{\alpha}} \leq \rho$ , where  $\rho = \max \rho_{\alpha, \tilde{\alpha}}$ .  $P_e(\cdot, \cdot)$  can be simplified to

$$\begin{aligned} P_e(\bar{\gamma}, \rho) &\leq \frac{L/K - 1}{\pi} \left( 1 + \frac{(1 - \rho^2) \bar{\gamma}}{8} \right)^{-2} (1 - \rho^2) \\ &\quad \times \left\{ 1 + \frac{|-\rho| \Gamma(3/2)}{1 + (1 - \rho^2) \bar{\gamma}/8} \right\}. \end{aligned} \quad (12)$$

As can be seen in (12),  $P_e(\cdot, \cdot)$  decreases with  $\bar{\gamma}$  for given  $\rho$ . Here, due to the fact that  $\bar{\gamma} = (L/K) E_s / N_0$ , it is worth mentioning that for given  $\rho$ ,  $P_e(\cdot, \cdot)$  in (12) can still retain low, properly balancing  $L$  and  $K$  for the  $K$ -sparsity. Moreover, notice that  $\rho$  can decrease with the maximum separation of  $\alpha_\Delta = K$ , leading to decrease in  $P_e(\cdot, \cdot)$ .

#### V. ASYMPTOTIC RESULTS

We now consider four extreme cases to investigate asymptotic behaviors of the proposed scheme in terms of the average propagation error rate and explore the impacts of the  $K$ -sparsity and the correlated sub-carriers on it.

##### A. Case of high SNRs

For high SNRs, the average propagation error rates in (12) can be approximated as

$$\begin{aligned} P_e(\bar{\gamma}, \rho) &\approx c \frac{\tau - 1}{(1 - \rho^2)} \bar{\gamma}^{-2}, \\ &= \Theta((G(\tau, \rho) \text{SNR})^{-2}), \end{aligned} \quad (13)$$

where  $\Theta(\cdot)$  is Big Theta notation,  $c$  denotes a scaling constant and  $\text{SNR} = E_s / N_0$ .

It is worth pointing out from (13) that the achievable diversity order is found to be two at high SNRs. Also, the power gain obtained is  $G(\tau, \rho) = \tau \sqrt{(1 - \rho^2)/(\tau - 1)}$ . This indicate that at high SNRs, a proper choice of  $\tau (= L/K)$  can enhance the power gain for a given  $\rho$ . The power gain further increase, minimizing  $\rho$  with the choice of the maximum  $\alpha_\Delta$  for given  $L$ . Such flexibility is not available in the classical OFDM.

##### B. Case of large sparsity: $L/K \gg 1$

Let  $K = \tau^{-1} L$  be an integer with a fixed  $\tau^{-1} \ll 1$ . As  $L$  grows, consider very large  $\tau (= L/K)$  when  $K \ll L$ . In this asymptotic case, the power gain achieved in (13) can be approximated as

$$G(\tau, \rho) \approx \sqrt{\frac{(1 - \rho^2)L}{K}}.$$

We can asymptotically observe from this the trade-off between  $G(\cdot, \cdot)$ ,  $\tau$ , and  $\rho$ . That is, as  $L$  increases with the fixed but very large sparsity ratio  $\tau$ , the power gain is proportional to  $\tau$  and scales inversely with  $\rho$ .

Interestingly, this observation raises the fundamental question that, as  $L$  grows, how large  $K$ -sparsity is worthy to make  $G(\cdot, \cdot)$  greater than or equal to the desired threshold ( $G_0$ ), i.e.,

$G(\cdot, \cdot) \geq G_0$ , with a positive  $\rho$ . We should properly find the sparsity ratio, satisfying

$$\tau \geq \frac{G_0^2}{1 - \rho^2}.$$

It asymptotically shows that *larger desired power gain, higher sparsity ratio  $\tau$  needs*, at high  $\rho$ .

### C. Case of highly correlated sub-carriers

When, in the ideal case, the orthogonality across sub-carriers is obtained having  $\rho = 0$ , then (12) can be simplified to

$$P_e(\bar{\gamma}) \leq \frac{\tau - 1}{\pi} \left(1 + \frac{\bar{\gamma}}{8}\right)^{-2}. \quad (14)$$

It can be seen that  $P_e(\cdot)$  relies on  $\tau = L/K$  and  $\bar{\gamma} = \tau E_s/N_0$ .

In practice, however, due to the presence of the hardware impairments (e.g., phase noise and frequency offsets), the non-orthogonality among the sub-carriers can be inevitable, providing  $\rho > 0$ . Accordingly, as  $\rho$  significantly grows in such frequency dispersive environments, (12) can be approximated, for a given  $\tau$ , as

$$P_e(\bar{\gamma}, \rho) \approx \frac{\tau - 1}{\pi} (1 - \rho^2) (1 + |\rho| \Gamma(3/2)). \quad (15)$$

It is worth mentioning that at large  $\rho$  (i.e.,  $\rho \rightarrow 1$ ), *the average propagation error rates are not limited by the SNR, but only  $\tau$  and  $\rho$ .*

In the MCIK with clusters, the level of  $\rho$  between *candidate* active sub-carriers (not between neighboring sub-carriers) can be effectively decreased by increasing  $\alpha_\Delta$ . In particular, consider the  $K$  clusters each comprising of  $N$  sub-carriers and we can select  $\alpha_\Delta \in \{1, \dots, K\}$ . The indices of sub-carriers at cluster  $k$  become  $\{\alpha_k, \alpha_k + \alpha_\Delta, \alpha_k + 2\alpha_\Delta, \dots, \alpha_k + (N - 1)\alpha_\Delta\}$ . The impact of  $\alpha_\Delta$  will be evaluated by simulations.

TABLE I  
SIMULATED CORRELATION COEFFICIENTS

$\sigma_{\text{rms}}$ (ns)	20 ns	50 ns	100 ns	200 ns
$\rho_{\alpha, \alpha+1}$	0.9995	0.9956	0.9817	0.927
$\rho_{\alpha, \alpha+\alpha_\Delta}$	0.9137	0.5878	0.3384	0.1761

## VI. SIMULATION RESULTS

We simulate the MCIK OFDM with the index separation under the independent Rayleigh fading channel per sub-carrier with BPSK, and QPSK modulation. The IEEE 802.11b multipath channel model is used, and the channel gains  $H_l$ 's among sub-carriers are assumed to be non-orthogonal leading to the cross-correlations  $\rho \in [0, 1)$ . The simulated values for  $\rho$  when  $L/K = 4$  are listed according to the rms delay spread in Table I. At the receiver, we employ the ML detection to estimate  $K$ -sparse signals.

Consider  $L = 64$  sub-carriers with various sparsity ratios:  $L/K \in \{4, \dots, 32\}$ . In order to evaluate the proposed scheme, the average propagation of the error rates (PER) are obtained and the average symbol error rates are simulated in comparison with the classical OFDM and the MCIK OFDM

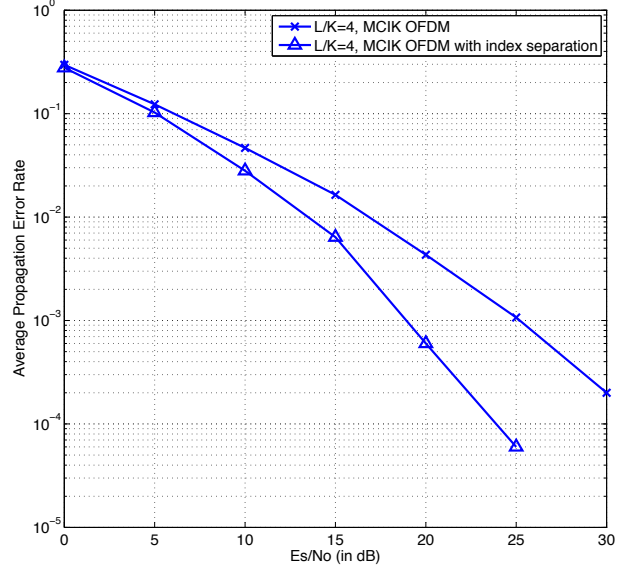


Fig. 1. The average propagation error rates of the MCIK-OFDM with the index separation are simulated with the maximum index separations,  $\alpha_\Delta = 16$ . In comparison, the general MCIK-OFDM without the index separation is depicted.

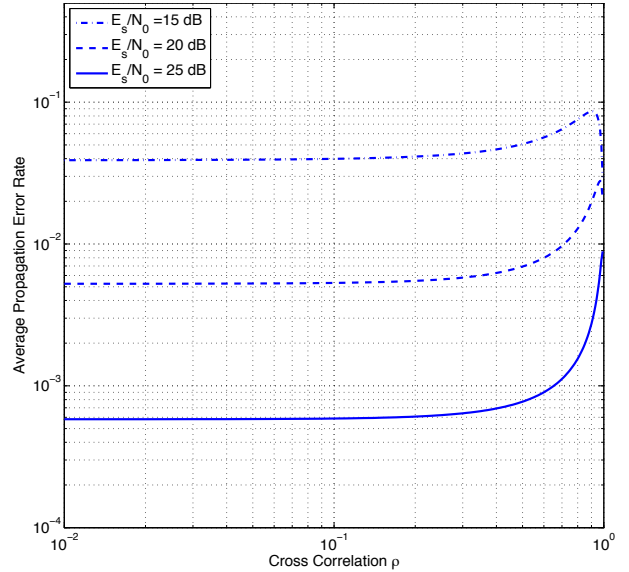


Fig. 2. The average propagation error rates are illustrated with the cross correlation. For the illustrations, we use that  $L = 64$ ,  $\tau = 4$  and  $E_s/N_0 \in \{15, 20, 25\}$  (dB).

without the index separation. Since large propagation of the errors affects all the error cases in demodulation, we also evaluate how large sparsity of the MCIK signals would be effective to reduce the error propagation in the presence of non-orthogonal sub-carriers.

Fig. 1 depicts the simulated average PER of the MCIK OFDM with the index separation. For the illustrations, BPSK is employed for modulation of non-zero data symbols with  $L = 64$ ,  $\tau = 4$ , and the maximum index separation  $\alpha_\Delta = 16$ . This figure illustrates the average PER decreases with

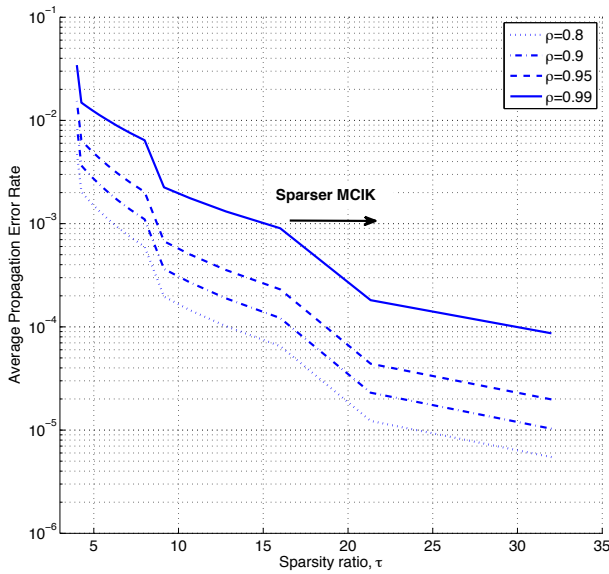


Fig. 3. The impact of the sparsity ratio on the average propagation error rates is illustrated at various cross correlations. We use when  $L = 64$ ,  $E_s/N_0 = 20\text{dB}$ , and  $\rho \in \{0.8, 0.9, 0.95, 0.99\}$ .

the diversity order of 2 at high SNRs. Compared to the MCIK-OFDM, this figure also illustrates that the average PER decreases faster when the index separation is maximized reducing the value for  $\rho_{\alpha, \tilde{\alpha}}$ . This validate our analysis in (13).

In Fig. 2, the average PER has been illustrated with the cross correlation  $\rho$  at several SNRs, i.e.,  $E_s/N_0 \in \{15, 20, 25\}$  (dB), for given  $L = 64$ ,  $\tau = 4$  and QPSK. From this figure, we can clearly observe the impact of  $\rho$  (relying on both the various index separation and the delay spread) on the average PER. Particularly, it can be seen in this figure that for given  $E_s/N_0 = 25$  dB and  $\tau = 4$ , the average PER remains stationary at low and moderate  $\rho$ 's (e.g.,  $\rho \leq 0.5$ ) and grows significantly at large  $\rho$ 's (e.g.,  $\rho \geq 0.5$ ). This observation validates the analysis in (15).

Fig. 3 illustrates the average PER of the MCIK OFDM with the index separation with respect to the sparsity ratios at various  $\rho$ 's. For this, we use when  $L = 64$ ,  $\rho \in \{0.8, 0.9, 0.95, 0.99\}$  and  $E_s/N_0 = 20$  dB. As observed from this figure, properly selecting the values of  $\tau$  guarantees the average PER to be less than or equal to a target value, given  $\rho$ . For example, with the desired value of  $10^{-3}$ , the average PER can remain less than or equal to  $10^{-3}$  by selecting  $\tau \geq 6$  when  $\rho \leq 0.8$ . Similarly, selecting  $\tau$  at the minimum ( $\tau \geq 8$ ) when  $\rho \leq 0.9$  satisfies the average PER requirement. Under the average PER requirement, *higher correlation, larger sparsity needs*.

In Fig. 4, we simulate the average symbol error rates of the MCIK OFDM with the index separation. For this, the BPSK and QPSK modulations are used with  $\tau = 2$  and  $\tau = 4$ , respectively. For comparison, the conventional OFDM and the MCIK OFDM with no index separation are also depicted. As observed, the MCIK OFDM with the index separation performs the best among them in a wide range of SNRs. For

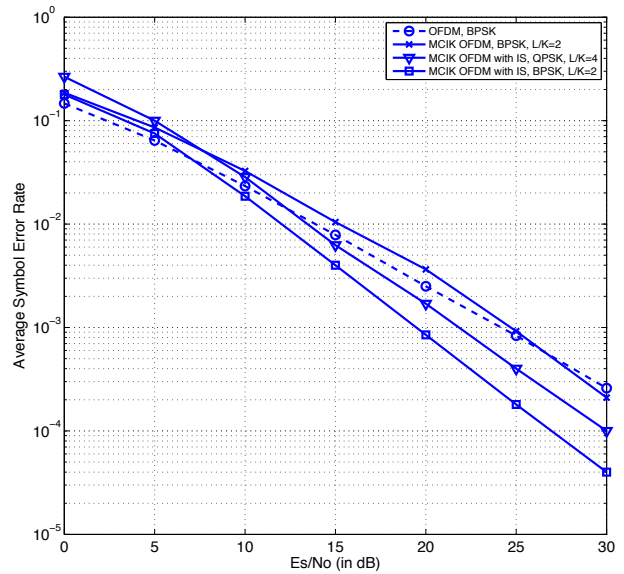


Fig. 4. Compared with the OFDM and the MCIK OFDM, the average symbol error rates of the proposed scheme is depicted. For the proposed MCIK OFDM with index separation, we use when  $L = 64$ , BPSK, QPSK, and  $\tau \in \{2, 4\}$ .

example, for given value of  $10^{-3}$ , the MCIK OFDM with the index separation and BPSK achieves the power gain of 5 dB over the OFDM scheme.

## VII. CONCLUSION

We proposed the MCIK with index separation for precoded OFDM that can exploit a power gain with a low-complexity receiver structure in highly correlated sub-carriers. Since the MCIK requires only a small subset of active sub-carriers, the number of demodulators used can be small, which allows to build a receiver at a low cost. In addition, one important feature of the proposed scheme was to develop the trade-off between the sparsity ratio, SNR, cross correlation, and performances. Fundamentally, this new feature is important for designs of future MCIK OFDM systems, answering how large sparsity of the multi-carrier index keying with the index separation should be in correlated sub-carriers.

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